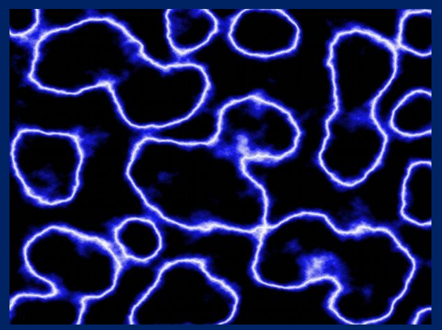
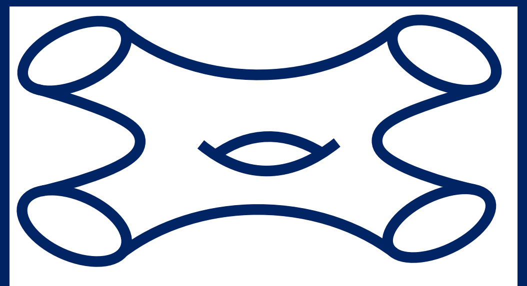
Type IIB at eight derivatives: Insights from Superstrings, Superfields and Superparticles

String Theory



Perturbation theory

String Scattering



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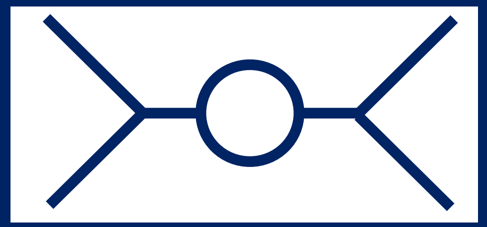


Off-shell SUGRA action

10D effective action

$$\int (\alpha')^3 R^4 \sqrt{g}\, \mathbf{d}^{10}x$$

Compactification on CY threefold X_3

4D effective action

$$\int \chi(X_3)\, R \sqrt{g}\, \mathbf{d}^4x$$

Applications

- Moduli stabilisation
- Model building
- Cosmology
- …

***Problem*:** Determining the α' expansion in 10-dimensional SUGRA action

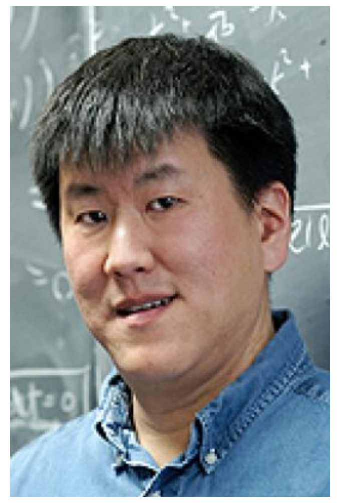
Noteworthy open challenges:

1. Eight-derivative couplings in the RR-sector
2. Role of supersymmetry in string kinematics
3. Consistency with string dualities and compactifications

Department of Applied Mathematics and Theoretical Physics

University of Cambridge

Based on ArXiv: 2205.11530 together with



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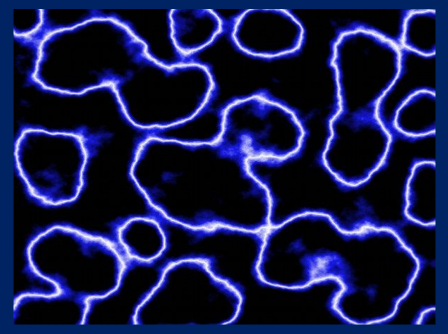

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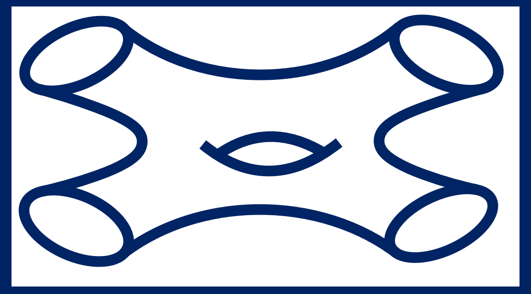

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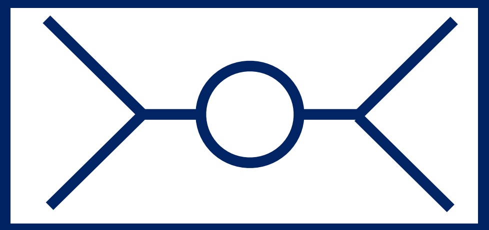

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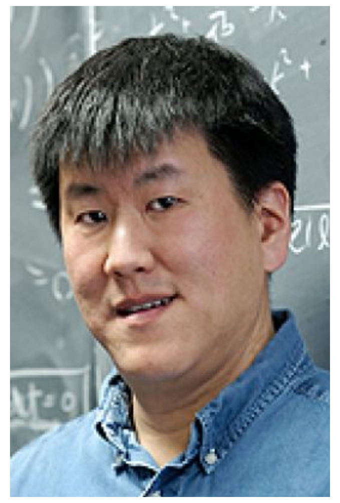

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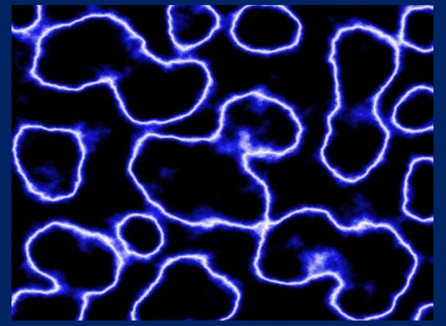

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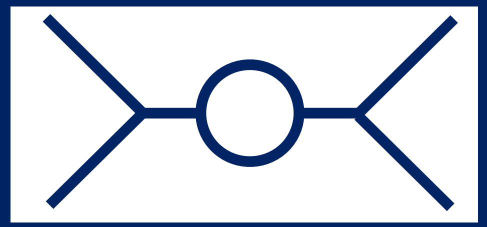

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Superstrings	10D Superfield	11D Superparticles
Tree and 1-loop string amplitudes up to six-points	String kinematics from compact superspace integrals	M-theory duality and derivation of SL$(2,\mathbb{Z})$ modular functions

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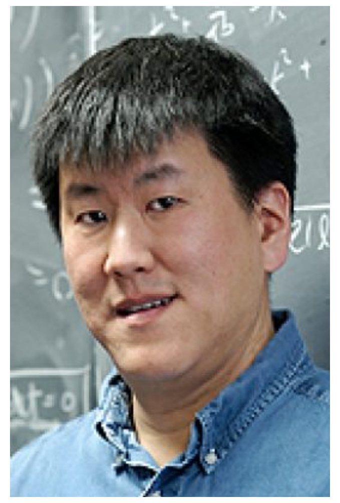

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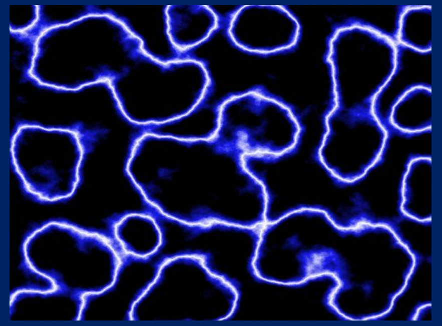

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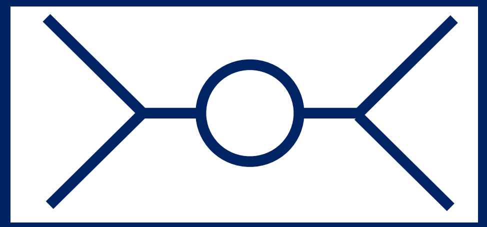

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Outline for this talk

1. Review of eight-derivative actions
2. Five-point kinematics for 3-forms
3. Structure of higher-point kinematics
4. Tests with lower dimensional SUSY

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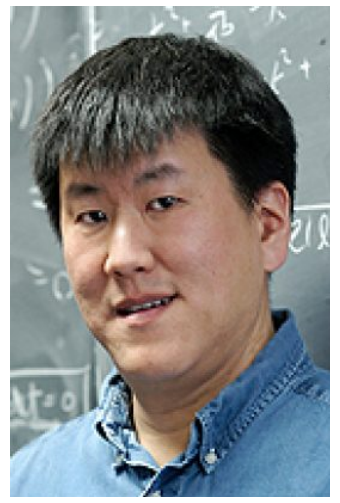

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Type IIB Supergravity and its α' Expansion

The α' expansion of Type IIB $\mathcal{N} = (2,0)$ SUGRA in 10D reads schematically (g_s expansion suppressed)

$$S_{IIB} = S_{IIB}^{tree} + (\alpha')^3 S_{IIB}^{(3)} + \sum_{n=4}^{\infty} (\alpha')^n S_{IIB}^{(n)}$$

No $(\alpha')^1$ **or** $(\alpha')^2$ to any order in g_s due to $\mathcal{N} = 2$ SUSY in 10D

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This talk: bulk corrections $S_{IIB}^{(3)}$ **at order $(\alpha')^3$ corresponding to 8-derivative operators**

$$S_{IIB}^{(3)} \sim \int \left\{ R^4 + R^3(G_3^2 + |G_3|^2 + \overline{G}_3^2 + F_5^2 + (\mathcal{P})^2 + \nabla\mathcal{P} + \ldots) + R^2(|\nabla G_3|^2 + (\nabla F_5)^2 + G_3^4 + \ldots) + R(G_3^6 + \ldots) + G_3^8 + (\nabla G_3)^4 + \ldots \right\}$$

in terms of the fields

$$\tau = C_0 + ie^{-\phi} \quad , \qquad \mathcal{P}_m = \frac{i}{2\mathrm{Im}(\tau)}\nabla_m \tau \quad , \qquad G_3 = \frac{1}{\mathrm{Im}(\tau)^{1/2}}(F_3 - \tau H_3)$$

Action is invariant under $\mathrm{SL}(2,\mathbb{Z})$ transformations

$$\tau \to \frac{a\tau + b}{c\tau + d} \quad , \qquad \mathcal{P} \to \frac{c\overline{\tau} + d}{c\tau + d}\mathcal{P} \quad , \qquad G_3 \to \left(\frac{c\overline{\tau} + d}{c\tau + d}\right)^{1/2} G_3$$

and a local $U(1)$ symmetry with $\mathrm{U}(1)$ charge assignments

$$Q_{U(1)}(\mathcal{P}) = 2 \quad , \quad Q_{U(1)}(G_3) = 1 \quad , \quad Q_{U(1)}(g_{MN}) = Q_{U(1)}(F_5) = 0$$

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The full quartic action

Quartic action completely determined [Policastro 0812.3138, Liu 1912.10974]

$$\mathcal{L}^{(3)} \supset f_0(\tau,\bar{\tau})\left(t_8 t_8 + \frac{1}{8}\epsilon_{10}\epsilon_{10}\right)\Big\{ R^4 + 6R^2(|\nabla\mathcal{P}|^2 + |\nabla G_3|^2) + \ldots \Big\}$$

in terms of the **Eisenstein series of weight 3/2** [Green hep-th/9706175]

$$f_0(\tau,\bar{\tau}) = \underbrace{2\zeta(3)\,\mathrm{Im}(\tau)^{3/2}}_{\text{Tree}} + \underbrace{\frac{2\pi^2}{3}\mathrm{Im}(\tau)^{-1/2}}_{\text{1-loop}} + \underbrace{\mathcal{O}\left(e^{-\mathrm{Im}(\tau)}\right)}_{\text{D-instantons}}$$

The kinematics is (mostly) fixed by the notion of **index structures t_8 and ϵ_{10}** as well as related to 12D covariance [Minasian 1506.06756]

Systematics of higher point functions

Partial 5- and 6-point results derived in **[1-4]** where one expects couplings to carry a **non-vanishing** total $U(1)$ charge

$$G_3^2 R^3 \quad , \quad (\nabla \mathscr{P})\overline{\mathscr{P}}^2 R^2 \quad , \quad G_3^2 \,|\nabla G_3|^2 R \quad , \quad G_3^4 R^2 \quad , \quad \ldots$$

References

[1]: Richards 0807.2421
[2]: Richards 0807.3453
[3]: Liu, Minasian 1304.3137
[4]: Liu, Minasian 1912.10974
[5]: Green et al. hep-th/9808061
[6]: Green et al. 1904.13394
[7]: Green et al. hep-th/9710151
[8]: Boels 1204.4208
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Their coefficients are determined by supersymmetry **[5]** and given by $\mathrm{SL}(2,\mathbb{Z})$**-covariant modular forms** (see e.g. **[6]**)

$$f_w(\tau,\bar{\tau}) = \sum_{(\hat{l}_1,\hat{l}_2)\neq(0,0)} \frac{\mathrm{Im}(\tau)^{\frac{3}{2}}}{(\hat{l}_1+\tau\hat{l}_2)^{\frac{3}{2}+w}(\hat{l}_1+\bar{\tau}\hat{l}_2)^{\frac{3}{2}-w}} \quad , \quad f_w\left(\frac{a\tau+b}{c\tau+d},\frac{a\bar{\tau}+b}{c\bar{\tau}+d}\right) = \left(\frac{c\tau+d}{c\bar{\tau}+d}\right)^w f_w(\tau,\bar{\tau}) \quad , \quad Q_{U(1)}(f_w) = -2w \quad , \quad \bar{f}_w = f_{-w}$$

so that schematically

$$\mathscr{L}^{(3)} = f_{12}\,\Lambda^{16} + \ldots + f_4\, G_3^8 + \ldots + f_1\, G_3^2 R^3 + f_0\,(R^4 + |G_3|^2 R^3 + \ldots) + f_{-1}\,\overline{G}_3^2 R^3 + \ldots + f_{-12}\,(\Lambda^*)^{16}$$

The terms Λ^{16} of maximal U(1)-charge involving the dilatino Λ where explicitly derived in **[7]**.

The kinematics for $P \geq 5$ amplitudes in the effective action remains largely unfixed.

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The role of $U(1)$ violation

Contact terms in $\mathscr{L}^{(3)}$ are best organised **by their $U(1)$ charges.**

P-point amplitudes carry a maximal $U(1)$ charge **[8,6]** (derived from 10D superamplitudes **[9]**)

$$|Q_{U(1)}| \leq 2(P-4)$$

Amplitudes saturating this bound, are called **maximally $U(1)$-violating (MUV) [8,6]** and have special properties:

- free of massless poles (“what you see is what you get“),
- uniquely fixed by supersymmetry,
- kinematically determined by a linearised superfield.

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Five-point Kinematics for 3-forms

The 1-loop (tree) amplitudes were computed in **[1,2]** (**[3,4,5]**) and the effective action was constructed in **[5]** (see also **[2]**)

$$\mathcal{L}_{R(\Omega_+)^4} = \alpha f_0(\tau,\bar\tau)\left[t_8t_8 - \frac{1}{4}\epsilon_8\epsilon_8\right]\left(R^4 + 6\,|\nabla G_3|^2 R^2 + 2\,|G_3|^2 R^3\right)$$

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$$\mathcal{L}_{G_3^2R^3+\text{c.c.}} = \alpha f_1(\tau,\bar\tau)\left[\frac{3}{4}t_8t_8G_3^2R^3 - \frac{1}{16}\epsilon_9\epsilon_9G_3^2R^3 - 3\cdot 4!\sum_i d_i\, G_3^{\mu\nu\lambda}\,G_3^{\rho\sigma\zeta}\,\tilde{Q}^i_{\mu\nu\lambda\rho\sigma\zeta}\right]+\text{c.c.}$$

$$\mathcal{L}_{CP-odd} = 9\cdot 2^4\,\alpha\, G_3 \wedge \left[f_0(\tau,\bar\tau)\, X_7(\Omega,\overline{G}_3) + f_1(\tau,\bar\tau)\, X_7(\Omega, G_3)\right]+\text{c.c.}$$

where we defined

$$\tilde{Q}^i_{\mu\nu\lambda\rho\sigma\zeta} = (R^3)_{\mu\nu\lambda\rho\sigma\zeta} \quad , \quad (d_1,\ldots,d_8) = 4\left(1,-\frac{1}{4},0,\frac{1}{3},1,\frac{1}{4},-2,\frac{1}{8}\right) \quad , \quad \alpha = \frac{(\alpha')^3}{3\cdot 2^{12}}$$

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Generalised geometry: couplings in $\mathscr{L}_{R(\Omega_+)^4}$ obtained from Riemann tensor with torsionful connection: $\Omega_\pm = \Omega \pm H_3$

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$$\mathscr{L}_{CP-odd} = 9\cdot 2^4\,\alpha\,G_3\wedge\left[f_0(\tau,\bar\tau)\,X_7(\Omega,\overline{G}_3) + f_1(\tau,\bar\tau)\,X_7(\Omega,G_3)\right]+\text{c.c.}$$

where we defined

$$\tilde{Q}^i_{\mu\nu\lambda\rho\sigma\zeta} = (R^3)_{\mu\nu\lambda\rho\sigma\zeta} \quad , \quad (d_1,\ldots,d_8) = 4\left(1,-\frac{1}{4},0,\frac{1}{3},1,\frac{1}{4},-2,\frac{1}{8}\right) \quad , \quad \alpha = \frac{(\alpha')^3}{3\cdot 2^{12}}$$

Generalised geometry: couplings in $\mathscr{L}_{R(\Omega_+)^4}$ obtained from Riemann tensor with torsionful connection: $\Omega_\pm = \Omega \pm H_3$

The presence of CP-odd couplings has been established in both type IIA **[6,7]** (odd in B_2) and type IIB **[8,5]** (even in B_2).

References

[1]: Peeters et al. hep-th/0112157

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Five-point Kinematics for 3-forms

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Guiding Questions for the remainder of this talk

1. Can we simplify these formulae? ➡ New index structures
2. How are the relative coefficients determined? ➡ SUSY
3. Are results consistent with SUSY compactifications? ➡ Dualities

References

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Intuition from Superparticle Amplitudes in 11 Dimensions

We compute supergravity amplitudes using a worldline approach parametrised by **Schwinger proper time [1], review [2]**

Famously: this string-inspired formalism has appeared as an efficient way to compute 1-loop QCD scattering amplitudes **[7]**

Compactified P-point superparticle amplitude in 11D SUGRA on a **2-torus** T^2 with $\mathrm{Vol}(T^2) = v_0$ **[3-5]**

$$\mathscr{A}_P(h, C_3) = \frac{1}{v_0}\int \frac{\mathrm{d}t}{t} \int \mathrm{d}^9 p\, \mathrm{e}^{-t\,\mathbf{p}^2} \sum_{l_1,l_2} \mathrm{e}^{-t\, g^{ab}\, l_a l_b}\, \mathrm{Tr}_S \left\langle \prod_{r=1}^{P} \int_0^t \mathrm{d}t^{(r)}\, V_r(h, C_3; t^{(r)}) \right\rangle \xrightarrow{v_0 \to 0} \text{Effective Vertices in 10D type IIB action}$$

- Integral over non-compact momenta: $\int \mathrm{d}^9 p\, \mathrm{e}^{-t\,\mathbf{p}^2}$
- Sum of KK/winding charges along T^2: $\sum_{l_1,l_2} \mathrm{e}^{-t\, g^{ab}\, l_a l_b}$
- Trace over fermion operators: Tr_S
- Integration over insertion points on the worldline: $\prod_{r=1}^{P} \int_0^t \mathrm{d}t^{(r)}$
- Vertex operators of 9D M-theory: $V_r(h, C_3; t^{(r)})$

Shown to contain information about **all string loop orders [6]**

Degrees of freedom **running in the loop** are the 9D supergraviton multiplet and KK/winding modes from the T^2

References

[1]: Strassler hep-ph/9205205

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Example: 4 graviton amplitude [3]

Kinematics from trace over fermion zero modes

$$K_{R^4} = \mathrm{Tr}(\,\theta\,\Gamma^{i_1 i_2}\theta\, \ldots\, \theta\,\Gamma^{i_{15}i_{16}}\theta\,)\, R_{i_1 i_2 i_3 i_4} \ldots R_{i_{13}i_{14}i_{15}i_{16}} = t_{16} R^4$$

$$v_0\, \mathscr{A}_4(h^4) = K_{R^4}\left(C v_0 + \frac{f_0(\tau,\bar\tau)}{\sqrt{v_0}} \right)$$

Zero winding $\hat{l}_1 = \hat{l}_2 = 0$ contributes to 11D M-theory action as $v_0 \to \infty$

Non-zero winding contributes in the limit $v_0 \to 0$

$$f_w(\tau,\bar\tau) = \sum_{(\hat{l}_1,\hat{l}_2)\neq(0,0)} \frac{\mathrm{Im}(\tau)^{\frac{3}{2}}}{(\hat{l}_1 + \tau\hat{l}_2)^{\frac{3}{2}+w}(\hat{l}_1 + \bar\tau\hat{l}_2)^{\frac{3}{2}-w}}$$

Winding charges of worldlines wrapping the T^2

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Takeaway messages:

1. One amplitude agrees with 4-point string scattering at
   - tree level **[8]**,
   - 1-loop **[9]**,
   - D-instanton level **[10]**.
2. Kinematics from higher-dim. tensors
3. Modular forms from torus winding

References

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5-point Contact Terms from Superparticles

Similarly for higher-point amplitudes involving the complex 3-form G_3 where at 5-points it is easy to derive

$$v_0 \mathscr{A}_{|G_3|^2 R^3} = \left[t_{18} - 18 t_{18}^{z\bar{z}}\right] |G_3|^2 R^3 \left(C\, v_0 - \frac{f_0(\tau,\bar{\tau})}{\sqrt{v_0}} \right) \quad , \quad v_0 \mathscr{A}_{G_3^2 R^3} = \frac{3}{2} t_{18} G_3^2 R^3 \frac{f_1(\tau,\bar{\tau})}{\sqrt{v_0}}$$

in terms of a new index structure

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- **No zero winding** contribution for $G_3^2\, R^3$ since no $U(1)$-violating terms in 11D M-theory **[4,5]**
- **Microscopic derivation** of $\mathrm{SL}(2,\mathbb{Z})$ modular forms f_w from integrating out KK-modes on the torus!

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Revised five-point effective action

$$\mathscr{L} = f_0\, t_{16}\, R^4 + f_0 \left[T(t_8, \epsilon_{10}) - t_{18} \right] |\, G_3\,|^2 R^3 + \frac{3}{2} \left(f_1\, t_{18} G_3^2 R^3 + f_{-1}\, t_{18} \overline{G}_3^2 R^3 \right) \qquad T(t_8, \epsilon_{10}) = 2\, t_8\, t_8 - \frac{1}{2} \epsilon_8\, \epsilon_8 - \frac{1}{3} \epsilon_9\, \epsilon_9$$

- terms $\sim \tilde{Q}^i$ found in **[1]** disappear making t_{18} the appropriate kinematical representation!
- MUV kinematics completely specified by t_{18} as expected from linearised superfield **[6]**
- non-MUV kinematics receives a piece $T(t_8, \epsilon_{10})$ from non-linear SUSY **[7,8]**

References

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Superstrings, Superparticles and … SUPERFIELDS

$$\mathscr{L}(G_3, \overline{G}_3, R) = f_0\, t_{16}\, R^4 + f_0 \left[T(t_8, \epsilon_{10}) - t_{18}\right] |\, G_3\,|^2 R^3 + \frac{3}{2}\left(f_1\, t_{18} G_3^2 R^3 + f_{-1}\, t_{18} \overline{G}_3^2 R^3\right)$$

Introduce supercharges $\Theta = \theta_1 + i\theta_2$ with Weyl spinors $\theta_1\,,\theta_2$ of $\mathrm{Spin}(1{,}9)$ to define a scalar superfield $\Phi(x,\Theta)$ with components **[4]**

$$\Phi = \sum_{r=0}^{8} \Theta^r \Phi^{(r)} = \tau + \Theta\Lambda + \Theta^2 G_3 + \Theta^3(\partial\psi + \ldots) + \Theta^4(R + \nabla F_5 + F_5^2 + |\, G_3\,|^2) + \ldots + \Theta^8(\nabla^4 \bar{\tau} + \ldots)$$

where we suppressed indices and 10D Γ-matrices. For instance, one recovers as above

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Kinematically, the superfield reproduces the previous results in the MUV sector!

References

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The effective action for MUV contact terms

Using linearised Type IIB superspace and superparticle methods, we derive higher order MUV contact terms as

$$\mathcal{L}^{MUV}(G_3, \overline{G}_3, R) = \sum_{w=0}^{4} c_w\, f_w(\tau, \bar{\tau})\, t_{16+2w}\, G_3^{2w}\, R^{4-w} + \mathrm{c\,.\,c\,.} \qquad c_w = \frac{2}{\sqrt{\pi}}\, \Gamma\!\left(\frac{3}{2} + w\right)$$

where the coefficients c_w are determined from superparticle amplitudes and consistent with SUSY **[8,5]** since

$$c_w f_w = 2^w\, \mathcal{D}_{w-1} \ldots \mathcal{D}_0\, f_0 \qquad \mathcal{D}_w f_w = i\left(\tau_2 \frac{\partial}{\partial\tau} - i\frac{w}{2}\right) f_w = \frac{3 + 2w}{4} f_{w+1}$$

These results match the 10D **superstring amplitudes** of **[5]** (using **[6,7]**) up to six-points!

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Application 1 — K3 reductions to SUGRA in 6 dimensions

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We extend results of **[Liu, Minasian 1304.3137, 1912.10974]** to RR-sector by focussing on factorised pieces:

$$\int_{K3} G_3^2 R^3 \supset G_3^2 R \int_{K3} R^2$$

As required by SUSY **[Lin et al. 1508.07305]**, we verify that the 3-point functions $|\, G_3\,|^2\, R, G_3^2\, R$ vanish.

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This analysis tests the coefficients of odd-odd sector couplings like $\epsilon_9 \epsilon_9$ and $\epsilon_8 \epsilon_8$ to which the Calabi-Yau threefold analysis (at 2-derivatives) is insensitive to!

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From reducing $f_0\, t_{16}\, R^4$, we can identify the correction to the Kähler potential **[Antoniadis et al. hep-th/9707013]**

$$K = K^{(0)} - 2\log\left[\mathscr{V} + \frac{\zeta}{4} f_0(\tau, \bar{\tau})\right], \quad \zeta = -\frac{\chi(X_3)}{2(2\pi)^3}, \quad K^{(0)} = -\log[-i(\tau - \bar{\tau})] - \log\left[-i\int_{X_3} \Omega \wedge \overline{\Omega}\right]$$

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Kinetic terms for B_2/C_2-axions

We also derived 4D kinetic terms, but issues remain! At NSNS tree level, we trivially agree with **[Grimm et al.: 1702.08404]**.

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$$= \frac{\zeta e^{K^{(0)}}}{\mathscr{V}^3}\left(-\frac{1}{4}\right)\left[(-6a_T - 2a_L) e^{-2\phi} \int_{X_3} H_3 \wedge \Omega \int_{X_3} H_3 \wedge \overline{\Omega} + (-4a_L) \int_{X_3} F_3 \wedge \Omega \int_{X_3} F_3 \wedge \overline{\Omega} + \ldots \right]$$

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We find that **our results are consistent** with the 4D perspective **provided** (equivalently for $H_3^2 R^3$ as well as $|G_3|^2 R^3$, $G_3^2 R^3$)

$$\alpha\int_{X_3}\left(t_{18} + \frac{2}{3}\delta_1\right) F_3^2 R^3 = -\frac{\zeta e^{K^{(0)}}}{4\mathcal{V}^3}\int_{X_3} F_3\wedge\Omega\int_{X_3} F_3\wedge\overline{\Omega}$$

Here, δ_1 is a potential backreaction effect entering in the MUV sector. Apart from that, **only the MUV kinematics t_{18} is relevant to determine V_ζ!**

Superstring amplitudes:

- Can be computed systematically
- Cumbersome to extract contact interactions
- Unintuitive representations for kinematical structures

Combination of the three strategies provides framework to efficiently extract full 8-derivative effective action!(?)

$$\mathscr{L} = f_0\, t_{16}\, R^4 + f_0 \left[T(t_8, \epsilon_{10}) - t_{18}\right] |\, G_3\,|^2 R^3 + \frac{3}{2}\left(f_1\, t_{18} G_3^2 R^3 + f_{-1}\, t_{18} \overline{G}_3^2 R^3\right)$$

Superparticle amplitudes:

- Kinematics encoded in higher-dim. index structures
- Combines NSNS+RR sector
- Explains/derives modular forms
- Captures more than the linearised superfield

Superfield approach:

- Kinematics encoded in superspace integrals
- Evidence for new non-linear terms
- Supersymmetry manifest

Outlook

- 10D dilaton couplings from string amplitudes
- Further tests with lower dimensional SUSY like 4D kinetic terms for B_2/C_2-axions
- Non-linear extension of superspace formalism involving G_3 and τ

String Phenomenology 2022